3843

OTS: 60-31,740

JPRS: 3843

9 September 1960

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- USSR -

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19990611 128

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JPRS: 3843
CSO: 4054-D

AN APPROACH TOWARD A THEORY OF NONPERTURBED SYSTEMS WITH THREE CHANNELS FOR AUTOMATIC COMPENSATION OF ACCELERATIONS DUE TO GRAVITY

- USSR -

[Following is a translation of an article by V. A. Bodner and V. P. Seleznev in the Russian-language periodical Izvestiya Akademii Nauk SSSR, OTN, Energetika i Avtomatika (Bulletin of the Academy of Sciences USSR, Department of Technical Sciences, Power Engineering and Automation), No. 1, Moscow, January-February 1960, pages 76-85.]

Controlled motion of a body is possible in case the coordinates of its position, velocity, and other navigational elements in respect to a determined coordinate system are known. The continuous determination of navigational elements is effected by navigational systems which can be based on the use of the properties of physical fields (the gravitational field, magnetic and electric fields, etc., or by bearings on celestial bodies. Nonperturbed systems, which are frequently combined with astronomical systems, occupy a special place among these systems. An astroinertial system in which the favorable properties of an astronomical system add a nonperturbed system are combined is examined in this article [1]. However, the author has not taken into account the peculiarities of nonperturbed systems with three channels of automatic compensation and has not evaluated the errors of such systems.

This article contains a discussion of certain problems of the theory of hydroinertial systems operating in three-dimensional space and an analysis of methodical errors.

1. Principles of the Construction of nonperturbed Systems.

The navigational elements in a nonperturbed system are determined on the basis of the integration of accelerations measured by three accelerometers with mutually perpendicular axes. The accelerations are measured in respect to an inertial coordinate system which has been selected in such a manner as to have no angular velocities relative to stellar space and whose origin can move under the action of gravitational forces. The accelerometers measure only the accelerations caused by the action of forces of drag and the thrust of the engines, but not of gravitational forces.

Let us select a coordinate system. The origin of the coordinates of the inertial system can coincide with the center of inertia of any celestial body which is moving in a gravitational field. In flight near the earth, an equatorial system, two axes of which coincide with the plane of the equator and the third axis with the axis of rotation of the earth, can be utilized as the inertial coordinate system. The chief shortcoming of such a system is the change in the position of its axes relative to stellar space due to the rotation of the earth.

For interplanetary flights it is convenient to make use of an elliptical system of coordinates x_0, y_0, z_0 (Figure 1) with its origin in the center of the sun. Two axes (x_0, y_0) lie in the plane of the earth's orbit (ecliptic), and the third axis is perpendicular to this plane. One of the axes (x_0) can be directed toward the first-magnitude star Spica, which is located near the plane of the ecliptic. The position of the flying apparatus in such a coordinate system is determined by rectangular coordinates x, y, z. It is also possible to make use of a spherical coordinate system with the same origin, positions in which are determined by the astronomical latitude gamma and longitude lambda with radius vector R.

The ecliptic coordinate system possesses great stability. Thus, for example, the change in the astronomical latitude amounts to 0.07 microradians per year and the change in the longitude is not more than 0.1 microradians per year.

The following forces are acting on the flying apparatus: thrust, drag, and gravity. The vector of the first two forces is designated by F and the vector of the force of gravity by G. The components of these forces on the axes of the rectangular coordinate system $x_0y_0z_0$ are equal to F_x, F_y, F_z and G_x, G_y, G_z respectively. Let us use x, y, z and x', y', z' to designate the coordinates and the velocities of the center of mass of the flying apparatus in the selected inertial coordinate system $x_0y_0z_0$.

Expressions for the components of the acceleration of the center of mass of the flying apparatus can be obtained from the equations of motion

$$mx'' = F_x - G_x, \qquad my'' = F_y - G_y, \qquad mz'' = F_z - G_z \qquad (1.1)$$

where m is the mass of the flying apparatus.

If we make use of the expressions for the accelerations of the flying apparatus

$$a_{xo} = \frac{F_x}{m}, \qquad a_{yo} = \frac{F_y}{m}, \qquad a_{zo} = \frac{F_z}{m} \qquad (1.2)$$

and introduce the corresponding accelerations from the gravitational forces

$$g_x = \frac{G_x}{m}, \qquad g_y = \frac{G_y}{m}, \qquad g_z = \frac{G_z}{m} \qquad (1.3)$$

then we find from equation (1.1),

$$a_{xo} = x'' + g_x, \quad a_{yo} = y'' + g_y, \qquad a_{zo} = z'' + g_z \qquad (1.4)$$

Accelerometers whose axes of sensitivity coincide with the axes $x_0y_0z_0$ will be used to measure the accelerations of (1.4). If motion occurs only under the action of gravitational forces ($F = 0$), then the accelerations of (1.4) vanish and the accelerometers will not show anything.

A hydroinertial system should include a stabilized platform, three accelerometers with mutually perpendicular axes of sensitivity, integrators, a computer, and feedback elements (Figure 2). The axes of sensitivity of the accelerometers form an orthogonal coordinate system xyz which should be parallel with the inertial coordinate system $x_0y_0x_0$. Due to the drifting of each of the gyroscopes, however, the xyz system of coordinates deviates from the $x_0y_0z_0$ system by the angles alpha, beta, gamma (Figure 3).

The integrators are necessary for integrating the accelerations and for obtaining the velocity and coordinates of position. The computer and the feedback system produce signals to compensate for accelerations from gravitational forces. A shortcoming of the hydroinertial system is the fact that it cannot independently eliminate the angles alpha, beta and gamma. External information is needed to eliminate them.

2. Equations of Motion of a Nonperturbed System. In order to derive the equations of motion of an inertial system it is necessary to make use of the relationship between the components of the accelerations along the $x_0y_0z_0$ and the xyz axes. The components of the accelerations a_x, a_y, a_z measured by the accelerometers are not equal to the components of acceleration of the center of the mass of the flying apparatus a_{x0}, a_{y0}, a_{z0}. The relationship between these components is given in the table of direction cosines:

	x_0
x	cos alpha cos beta
y	sin alpha sin gamma - cos alpha sin beta cos gamma
z	sin alpha cos gamma + cos alpha sin beta sin gamma

	y_0
x	sin beta
y	cos beta cos gamma
z	- cos beta sin gamma

	z_0
x	- sin alpha cos beta
y	cos alpha sin gamma + sin alpha sin beta sin alpha
z	cos alpha cos gamma- sin alpha sin beta sin gamma

If we make use of the fact that under real conditions the angles alpha, beta, and gamma are small, then the sines can be replaced by the angles themselves, the cosines can be assumed to be equal to unity, and the products of the angles can be neglected as quantities of the second order of smallness. Then the relation between the components of acceleration along the xyz and the $x_0y_0z_0$ axes will be:

$$a_x = a_{x0} + \text{Delta } a_{xr} \qquad (2.1)$$
$$a_y = a_{y0} + \text{Delta } a_{yr}$$
$$a_z = a_{z0} + \text{Delta } a_{zr}$$

where
Delta $a_{xr} = a_{y0}$ Beta $- a_{z0}x$, Delta $a_{yr} = a_{z0} - a_{x0}$ Beta, Delta $a_{zr} = a_{x0}x - a_{y0}$ Gamma (2.2)

The errors Delta a_{xg} Delta a_{yg}, and Delta a_{xg} occur due to drifting of the gyroscopes, as a result of which the axes of sensitivity of the accelerometers do not coincide with the axes of the inertial coordinate system.

The output signals of the accelerometers, as may be seen from the structural diagram in Figure 4, pass through the integrators with transfer numbers k_{1x}, k_{1y}, k_{1z}, k_{2x}, k_{2y}, k_{2z} into the computer which computes the compensating signals g_{xc}, g_{yc}, g_{zc}, to compensate for accelerations from gravitational forces.

The coordinates of position S_x, S_y, S_z and the velocities of the center of mass S_x', S_y', S_z' of the flying apparatus are calculated as a result of the operation of the inertial system.

From the structural diagram (Figure 4) we obtain the following equations of motion of each of the channels of the inertial system;

$$\left[(a_z - g_{xr})\frac{k_{1x}}{p} + x_0'\right]\frac{k_{2x}}{p} + x_0 = S_x \qquad (2.3)$$

$$\left[(a_y - g_{yr})\frac{k_{1y}}{p} + y_0'\right]\frac{k_{2y}}{p} + y_0 = S_y$$

$$\left[(a_z - g_{zr})\frac{k_{1z}}{p} + z_0'\right]\frac{k_{2z}}{p} + z_0 = S_z$$

where x_0, y_0, z_0 and x_0', y_0', z_0' are the initial values of the coordinates and the velocities of the flying apparatus.

It can be seen from the structural diaagram (Figure 4) that the signals from the output of the first integrators are the components of the absolute velocities of flight, while the signals ar the output of the second integrators are the components of the distance travelled.

Let us transform the equations of (2.3), for which purpose we shall select transfer numbers of the integrators from the conditions $k_{1x}k_{2x} = k_{1y}k_{2y} = k_{1z}k_{2z} = 1$. We shall find

$$\text{Delta } x'' = \text{Delta } g_x + \text{Delta } a_{xr} \qquad (2.4)$$
$$\text{Delta } y'' = \text{Delta } g_y + \text{Delta } a_{yr}$$
$$\text{Delta } z'' = \text{Delta } g_z + \text{Delta } a_{zr}$$

where
Delta $x = S_x - x$, Delta $y = S_y - y$, Delta $z = - z$ (2.5)
the errors of the inertial system,
Delta $g_x = g_x - g_{xr}$, Delta $g_y = g_y - g_{yr}$, Delta $g_z = g_z - g_{zr}$ (2.6)

the errors which characterize inaccurate compensation of accelerations due to gravitational forces.

If precise compensation of accelerations from gravitational forces is accomplished in an inertial system (Delta g_x = Delta g_y = Delta g_z = 0) and there is no drifting of the gyroscopes (Delta a_{xg} = Delta a_{yg} = Delta a_{zg} = 0), then it follows from equations (2.4) that the errors of the system will be determined by inaccuracy in setting the initial conditions.

Since automatic compensation of accelerations from gravitational forces cannot be precise and, in addition, there is drifting of the gyroscopes, different errors do occurr in an inertial system, and analysis of which is given below.

3. Errors in a System of Automatic Compensation for Accelerations Due to Gravitational Forces. The errors in a system of this type occur because the compensation signals g_{xc}, g_{yc}, g_{zc}, generated in the computer are not equal to the components of acceleration g_x, g_y, g_z. These errors are fed into an inertial system and, afterpassing through a closed cycle, are again fed into the computer. If the inertial system is stable, the errors of automatic compensation will be damped. In the opposite case they will grow, which will give rise to an inaccurate determination of position.

In order to generate the compensation signals it is essential to know the analytical expressions for the components of acceleration g_x, g_y, g_z which are caused by gravitational forces of celestial bodies acting on the flying apparatus. if m_i, x_i, y_i, z_i (i = 1, 2, 3, ..., n) the masses and coordinates of the centers of masses of celestial bodies that are accelerating the flying apparatus, the components of the accelerations will be

$$g_x = \text{Sigma}_{i=1}^{n} fm_i \frac{x-x_i}{R_i^3}, \quad g_y = \text{Sigma}_{i=1}^{n} fm_i \frac{y-y_i}{R_i^3}, \quad g_z = \text{Sigma}_{i=1}^{n} fm_i \frac{z-z_i}{R_i^3} \qquad (3.1)$$

where

$$R_i = \sqrt{(x-x_i)^2 -- (y-y_i)^2 -- (z-z_i)^2} \qquad (3.2)$$

and f is the gravitational constant, which is equal to 6.67×10^{-8} cgs.

The computer should generate compensation signals g_{xc}, g_{yc}, g_{zc} in accordance with the formulas (3.1) and (3.2) in which it is necessary to substitute the measured inertial coordinate system S_x, S_y, S_z in place of the actual coordinates of the flying apparatus x, y, z. Consequently, the computer should calculate the compensation signals in accordance with the formulas

$$g_{xr} = \text{Sigma}_{i=1}^{n} fm_i \frac{S_x - x_i}{\bar{R}_i^3}, \quad g_{yr} = \text{Sigma}_{i=1}^{n} fm_i \frac{S_y - y_i}{\bar{R}_i^3}, \quad g_{zr} = \text{Sigma}_{i=1}^{n} fm_i \frac{S_z - z_i}{\bar{R}_i^3} \qquad (3.30)$$

$$\bar{R}_i = \sqrt{(S_x - x_i)^2 + (S_y - y_i)^2 + (S_z - z_i)^2} \qquad (3.4)$$

In order to make the calculations it is also necessary to know the masses m_i and the coordinates x_i, y_i, z_i of the celestial bodies, which can be obtained with great accuracy.

If the differences Delta g_x, Delta g_y, Delta g_z are formed by substituting the expressions (3.1) and (3.3) into (2.6) and they are decomposed into aeries by the increments Delta x, Delta y, Delta z then, limiting ourselves to the linear terms, we find

$$\Delta g_x = -\sum_{i=1}^{n} \frac{fm_i}{R_i^3}\left\{\left[1-3\left(\frac{x-x_i}{R_i}\right)^2\right]\Delta x - 3\frac{x-x_i}{R_i^2}\left[(y-y_i)\Delta y + (z-z_i)\Delta z\right]\right\} \qquad (3.5)$$

$$\Delta g_y = -\sum_{i=1}^{n} \frac{fm_i}{R^3}\left\{\left[1-3\left(\frac{y-y_i}{R_i}\right)^2\right]\Delta y - 2\frac{y-y_i}{R_i^2}\left[(x-x_i)\Delta x + (z-z_i)\Delta z\right]\right\}$$

$$\Delta g_z = -\sum_{i=1}^{n} \frac{fm_i}{R_i^3}\left\{\left[1-3\left(\frac{z-z_i}{R_i}\right)^3\right]\Delta z - 3\frac{z-z_i}{R_i^2}\left[(x-x_i)\Delta x + (y-y_i)\Delta y\right]\right\}$$

Substituting (3.5) into (2.4), we obtain (3.6)

$$(p^2 + n_x^2)\Delta x - A\Delta y - B\Delta z = \Delta a_{xr}$$

$$-A\Delta x + (p^2 + n_y^2)\Delta y - C\Delta z = \Delta a_{yr}$$

$$-B\Delta x - C\Delta y + (p^2 + n_z^2)\Delta z = \Delta a_{zr}$$

where (3.7)

$$n_x^2 = \sum_{i=1}^{n} \frac{fm_i}{R_i^3}\left[1-3\left(\frac{x-x_i}{R_i^2}\right)\right], \quad n_y^2 = \sum_{i=1}^{n} \frac{fm_i}{R_i^3}\left[1-3\frac{(y-y_i)^2}{R_i^2}\right.,$$

$$n_z^2 = \sum_{i=1}^{nn} \frac{fm_i}{R_i^3}\left[1-3\frac{(z-z_i)^2}{R_i^2}\right]$$

$$A = 3 \sum_{i=1}^{n} \frac{f m_i}{R_i^3} \frac{(x-x_i)(y-y_i)}{R_i^2}, \quad B = 3 \sum_{i=1}^{n} \frac{f m_i}{R_i^3} \frac{(x-x_i)(z-z_i)}{R_i^2}, \tag{3.8}$$

$$C = 3 \sum_{i=1}^{n} \frac{f m_i}{R_i^3} \frac{(y-y_i)(z-z_i)}{R_i^2}$$

$$p = \frac{d}{dt}$$

Consequently, the errors in the inertial system which arise out of inaccurate automatic compensation of accelerations due to gravitational forces are determined by a system of linear differential equations with variable coefficients.

Let us examine the behavior of the system (3.6) under the condition that there is no drifting of gyroscopes. The behavior of the inertial system in respect to the errors Delta x, Delta y, Delta z is determined by the type of solution of the equations (3.6). It is obvious that if the system is stable, the automatic compensation errors will vanish. In case the system is unstable, the errors will grow.

Let us study the stability of the system of automatic compensation for accelerations. For this purpose we shall examine the partial case, assuming that the coefficients of the system (3.6) change slowly in comparison with the time required for processes to take place in the automatic compensation system. Making use of this assumption and considering the coefficients of the system (3.6) to be constants for the sake of simplicity, we can judge its stability on the basis of the properties of the characteristic determinant

$$\Delta = \begin{vmatrix} p^2 + n_x^2 & -A & -B \\ -A & p^2 + n_y^2 & -C \\ -B & -C & p^2 + n_z^2 \end{vmatrix} \tag{3.9}$$

If this determinant is expanded and set equal to zero, we shall obtain the characteristic equation of the system (3.6)

$$p^6 + (n_x^2 + n_y^2 + n_z^2) p^4 + (n_x^2 n_y^2 + n_x^2 n_z^2 + n_y^2 n_z^2 - A^2 - B^2 - C^2) p^2 + n_x^2 n_y^2 n_z^2 - A^2 n_z^2 - B^2 n_y^2 - C^2 n_x^2 - 2ABC = 0 \tag{3.10}$$

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It follows from this equation that the system of automatic compensation of accelerations due to gravitational forces is unstable, as equation (3.10 lacks terms with odd powers of the operator $p = d/dt$.

Lt us transform equation (3.10), for which we shall evaluate its coefficients by making use of formulas (3.7) and (3.8). At the same time, we shall assume for the sake of simplicity that the flying apparatus is in the gravitational field of one celestial body, for example, the sun. In case it is essential to take into account the gravitational forces of several celestial bodies, one may introduce the concept of an equivalent celestial body, for which $g/R = \sum_{i=1}^{n} fm_i/R_i^3$.

We have (3.11)

$$n_x^2 + n_y^2 + n_z^2 = 0$$

$$n_x^2 n_y^2 + n_x^2 n_z^2 + n_y^2 n_z^2 - A^2 - B^2 - C^2 = -3 \frac{g^2}{R^2}$$

$$n_x^2 n_y^2 n_z^2 - A^2 n_z^2 - B^2 n_y^2 - C^2 n_x^2 - 2ABC = -2 \frac{g^3}{R^3}$$

where

$$g = \sqrt{g_x^2 + g_y^2 + g_z^2}$$

Substituting (3.11) into (3.10) and making the transformations, we obtain

$$(p^2 + \Omega^2)^2 (p^2 - 2 \Omega^2) = 0 \qquad (3.12)$$

where

$$\Omega^2 = g/R$$

It follows from this that the characteristic equation (3.12) has two double imaginary roots and two real roots, one of which is positive. Consequently, the general motion of the automatic compensation system consists of three motions: harmonic oscillations with a period of

$T_1 = 2 \pi / \Omega = 2 \Omega \sqrt{R/g}$; oscillations with the same period and an amplitude which grows proportionally with time; and finally, and aperiodic, exponentially growing motion with a time constant

$T_2 = 1/\sqrt{2 \Omega} = T_1/2\sqrt{2 \pi}$.

For a more detailed examination of the character of the motion of the system, let us examine the flight of a flying apparatus in the gravitational field of the sun in the direction of the y_0 axis. In this case $R = y$, $x = z = 0$, $g_x = g_z = 0$, $g_y = g$ and the equations (3.6) have the form

$$(p^2 + \Omega^2)\,\Delta x = \Delta a_{xr} \qquad (3.13)$$

$$(p^2 - 2\,\Omega^2)\,\Delta y = \Delta a_{yr}$$

$$(p^2 + \Omega^2)\,\Delta z = \Delta a_{zr}$$

It follows from this that in directions perpendicular to the resultant vector of the gravitational forces, the errors Delta x and Delta z change harmonically with the amplitude, which depends on the initial conditions and the period of oscillation mentioned above. In the direction of the vector of the gravitational forces, the error Delta y grows exponentially. If there is a connection between the channels, as shown above, a third motion appears with oscillations of increasing amplitude.

We note an analogy between the character of the change in the errors of the inertial system of navigation and the motion of a satellite. The period of change in the errors (oscillation period) of the inertial system T

$$T_1 = 2\,\pi\sqrt{R/g}$$

is equal to the period of a satellite flying in a circular orbit with radius R around a celestial body with the first cosmic velocity

$$V_1 = R\,\Omega = \sqrt{gR} \qquad (3.14)$$

When navigating at the surface of the earth the period of change in the errors Delta x and Delta z is equal to $T_1 = 84.4$ minutes and is called the Schuler period.

The error in the automatic compensation of accelerations due to gravitational forces Delta y in the direction of the resultant vector of the gravitational forces changes exponentially with the constant of time

$$T_2 = \frac{1}{\sqrt{2}}\sqrt{R/g},$$

which is $1/2\sqrt{2}\,\pi$ of the period of the inertial system. When the initial conditions are nonzero, the error grows continually, which characterizes the instability of the automatic compensation system. Consequently, switching on the automatic compensation channel of an inertial system in which the axis of the accelerometer coincides with the direction of the vector of the gravitational forces makes the system absolutely unstable.

The analogy between the inertial system and the motion of a satellite can be continued. We note in particular that the character of the change in the error Delta y corresponds with the change in the radius vector R of a satellite which is moving away from a celestial

body (for example, from the earth) with the second cosmic velocity

$$V_2 = \sqrt{2} \text{ Omega } R = \sqrt{2gR} \qquad (3.15)$$

Indeed, only when the flying apparatus (satellite) is moving at the second cosmic velocity does it overcome the attraction of the celestial body and move away from it freely. In the inertial system the signal in the automatic compensation channel which measures the coordinate Delta y fully compensates for the acceleration of gravity; therefore the inertial system is operating under conditions analogous to the physical absence of gravitational force.

The analogy between the motions of an inertial system and a satellite can be used for simulating the motion of a satellite.

In order to evaluate the effect of the initial error Delta y_0 and the altitude of flight above the earth on the magnitude of the error of the inertial system Delta y, determined by means of the second equation of (3.13) with a zero right-hand side, the corresponding graphs of Delta y/Delta y_0 = f(t) are presented in Figure 5. The curves 1, 2, and 3 correspond to flight altitudes of 0, 1,000, and 2,000 kilometers. The error accumulates more slowly with increased altitude, since the effect of gravitational forces decreases.

The properties of an inertial system which were studied here correspond not only to the special case of flight along a straight line, but also to arbitrary flight if the axis of sensitivity of one of the accelerometers is held in the direction of the vector of the gravitational forces. Under terrestrial conditions, this means directing the axis of sensitivity of one of the accelerometers along the vertical direction and the axes of sensitivity of the other two accelerometers in the horizontal plane. The errors in the channel which measures the altitude of flight will grow exponentially with zero initial conditions, while the errors in the channels which measure the coordinates of position will grow harmonically with the M. Schuler period.

When a flying apparatus moves along an arbitrary path, all the channels of a system of automatic compensation for accelerations due to gravitational forces are unstable, as shown by equations (3.6). When the initial conditions are nonzero, the errors in all three channels have terms which grow with time in addition to the oscillatory terms.

The instability of inertial systems with automatic compensation for accelerations due to gravitational forces cannot constitute a criterion of their inapplicability. Errors will not exist under zero initial conditions, and in case initial errors do exist, their growth is slow. In this case, the time of growth is comparable with the period of the system. Moreover, the system can be made stable by introducing additional links or by feeding additional external information into the system [2], for example, from astronomical navigation systems.

One can formulate the following theorems on the basis of what has been set forth here.

Theorem 1. Automatic compensation for accelerations due to gravitational forces in all three channels of an inertial system will lead to the characteristic equations

$$(p^2 + \text{Omega}^2)^2(p^2 - 2\ \text{Omega}^2) = 0 \tag{3.16}$$

for the errors in each of the channels.

Theorem 2. The period of oscillation of the errors of an inertial system is equal to the period of a satellite moving in a circular orbit with radius R around a celestial body at the first cosmic velocity $V_1 = \sqrt{gR}$.

Theorem 3. The constant of the time of growth of the errors of an inertial system is equal to the constant of the time of motion of a satellite which is moving away from a celestial body with the second cosmic velocity

$$V_2 = \sqrt{2gR}.$$

A method of "freezing" the coefficients is used in studying the equations (3.6) and as a result we obtain a system with constant coefficients. This assumption can be considered admissable for small intervals of time comparable with the period of the oscillations of an inertial system and the conclusions given above are justified. For large intervals of time it is essential to examine equations with variable coefficients. In this case the quantities T_1 and T_2 become variables, thus the period of oscillations and the constant of time lose their usual sense.

4. Instrumental Errors. Instrumental errors in an inertial system consist in drifting of the gyroplatform relative to the inertial coordinate system, errors in the accelerometers, the integrators, the computer, et cetera. Drifting of the gyroplatform consists of errors of two types: the errors linked with rotation of the xyz coordinate system relative to the inertial $x_0y_0z_0$ coordinate system and the errors generated in a closed compensation system.

An idea of the effect of instrumental errors on the accuracy of an inertial system can be obtained from the following considerations.

For Delta a_{xz}, ..., Delta b_{x1}, ..., Delta b_{x2}, ... which are the errors of the accelerometer, the first and second integrators respectively, equation (2.3) which determines the calculated coordinates acquires the form

$$\left[(x'' + \text{Delta } g_x + \text{Delta } a_{xr} + \text{Delta } a_{xa})\frac{k_{2x}}{p} + \text{Delta } b_{x1} + x_0'\right]\frac{k_{2x}}{p} + \text{Delta } b_{x2} + x_0 = S_x \tag{4.1}$$

Continuation of (4.1)

$$\left[\left(y'' + \text{Delta } g_y + \text{Delta } a_{yr} + \text{Delta } a_{ya}\right)\frac{k_{1y}}{p} + \text{Delta } b_{y1} + y_0'\right]$$

$$\frac{k_{2y}}{p} + \text{Delta } b_{y2} + y_0 = S_y$$

$$\left[\left(z'' + \text{Delta } g_z + \text{Delta } a_{zr} + \text{Delta } a_{za}\right)\frac{k_{1z}}{p} + \text{Delta } b_{z1} + z_0'\right]$$

$$\frac{k_{2z}}{p} + \text{Delta } b_{z2} + z_0 = S_z$$

It follows from this that instrumental errors in the elements of an inertial system have a direct effect on the accuracy with which the coordinates of position are determined. Instrumental errors will reach large magnitudes when an inertial system is operated over protracted periods of time.

The principal methods for reducing instrumental errors are essentially reducing the errors of the elements of the inertial system and correcting the system by introducing external navigational information. In the latter case the corrections can be made discretely, not continually.

In concluding, we note that an inertial system with three channels of automatic compensation for accelerations due to gravitational forces is unstable when a flying apparatus is moving arbitrarily and that the errors in this system will grow when the initial conditions are nonzero. In order to eliminate the errors it is necessary to correct the system continuously or discretely by feeding external navigational information into it. In this case the inertial system fulfills the role of a memory device in addition to its basic role.

The analogy between the changes in the errors of an inertial system with three automatic compensation channels and the motion of a satellite can be used in simulating the motion of a satellite.

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FIGURE APPENDIX

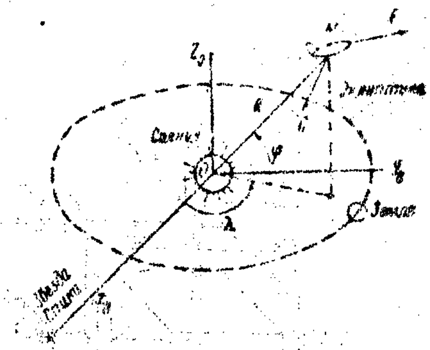

Figure 1.

FIGURE APPENDIX

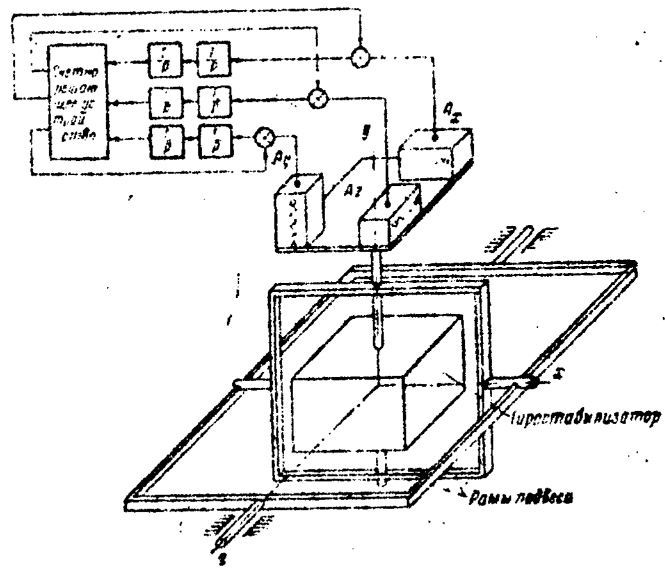

Figure 2.

FIGURE APPENDIX

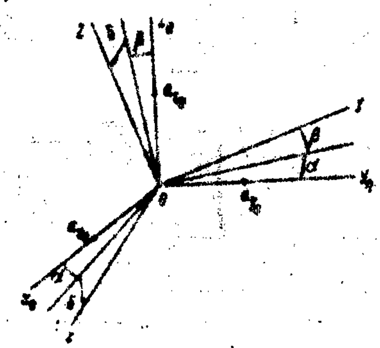

Figure 3.

FIGURE APPENDIX

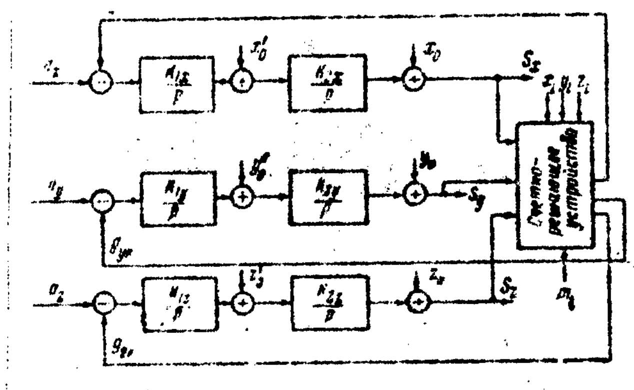

Figure 4.

FIGURE APPENDIX

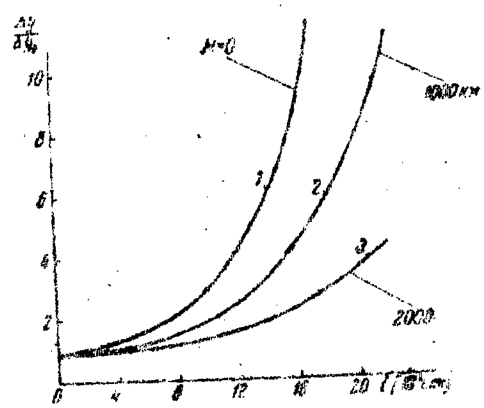

Figure 5.

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